Total Marks : $\mathbf{3 0}$
DPP No. 19 to 24
Max. Time : 30 min.

## Solution

## DPP NO. - 19

2. $\ln (A) \quad x_{f}-x_{i}$
$0-x=-x=-v e$
So average velocity is -ve .
3. From the graph ; we observe that slope is non-zero positive at $t=0$ \& slope is continuously decreasing with time and finally becomes zero. Hence we can say that the particle starts with a certain velocity, but the motion is retarded (decreasing velocity)
4. Suppose particle strikes wedge at height 'S' after time $t$. $S=15 t-\frac{1}{2} 10 t^{2}=15 t-5 t^{2}$. During this time distance travelled by particle in horizontal direction $=5 \sqrt{3} \mathrm{t}$. Also wedge has travelled travelled extra distance

$x=\frac{S}{\tan 30^{\circ}}=\frac{15 t-5 t^{2}}{1 / \sqrt{3}}$
Total distance travelled by wedge in time
$t=10 \sqrt{3} t .=5 \sqrt{3} t+\sqrt{3}\left(15-5 t^{2}\right)$
$\Rightarrow \mathrm{t}=2 \mathrm{sec}$.

## Alternate Sol.

(by Relative Motion)

$\mathrm{T}=\frac{2 \mathrm{u} \sin 30^{\circ}}{\mathrm{g} \cos 30^{\circ}}=\frac{2 \times 10 \sqrt{3}}{10} \times \frac{1}{\sqrt{3}}=2 \mathrm{sec}$.
$\Rightarrow \mathrm{t}=2 \mathrm{sec}$.
5.


As given

$$
\begin{aligned}
& \left(V_{A}-V_{B}\right) \propto X_{A}-x_{B} \\
& \left(V_{A}-V_{B}\right)=K\left(x_{A}-x_{B}\right)
\end{aligned}
$$

when $x_{A}-x_{B}=10$ We have $V_{A}-V_{B}=10$
We get

$$
\begin{align*}
& 10=K 10 \Rightarrow K=1 \\
& \Rightarrow V_{A}-V_{B}=\left(x_{A}-x_{B}\right) . \tag{1}
\end{align*}
$$

Now Let
$x_{A}-x_{B}=y$
On differentiating with respect to ' $t$ ' on both side.
$\Rightarrow \frac{\mathrm{dx}_{\mathrm{A}}}{\mathrm{dt}}-\frac{\mathrm{dx}}{\mathrm{B}} \mathrm{dt}=\frac{\mathrm{dy}}{\mathrm{dt}} \Rightarrow \mathrm{V}_{\mathrm{A}}-\mathrm{V}_{\mathrm{B}}=\frac{\mathrm{dy}}{\mathrm{dt}}$
$\Rightarrow$ Using (1) , (2), (3)
We get $\quad \frac{d y}{d t}=y$

Here y represents sepration between two cars
$\Rightarrow \int_{10}^{20} \frac{d y}{y}=\int_{0}^{t} d t \Rightarrow\left[\log _{e} y\right]_{10}^{20}=t$
$t=\left(\log _{\mathrm{e}} 2\right)$ sec $\quad$ Required Answer.


Alter. (Assume to be at rest)
$V \propto s$
$V=k s$
$V=10, s=10, k=1$

$$
\frac{\mathrm{ds}}{\mathrm{dt}}=\mathrm{s} \quad \int_{10}^{20} \frac{\mathrm{ds}}{\mathrm{~s}}=\int_{0}^{\mathrm{t}} \mathrm{dt}
$$

6 to 8. At $t=2 \sec \quad(t=2 \sec i j)$
$v_{x}=u_{x}+a_{x} t=0+10 \times 2=20 \mathrm{~m} / \mathrm{s}$
$v_{y}=u_{y}+a_{y} t=0-5 \times 2=-10 \mathrm{~m} / \mathrm{s}$
$v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(20)^{2}+(-10)^{2}}=10 \sqrt{5} \mathrm{~m} / \mathrm{s}$
From $t=0$ to $\mid \mathrm{S}=4 \mathrm{sec}$
$x=\left[\frac{1}{2}(10)(2)^{2}\right]_{(0 \rightarrow 2)}+\left[(10 \times 2) 2-\frac{1}{2}(10)(2)^{2}\right]_{(2 \rightarrow 4)}$
$x=40 \mathrm{~m}$
$y=\left[-\frac{1}{2} 5(2)^{2}\right]_{(0 \rightarrow 2)}-\left[\left(10(2)-\frac{1}{2}(10)(2)^{2}\right]_{(2 \rightarrow 4)}\right.$
$y=-10 m$
Hence, average velocity of particle between $t=0$ to $t=4 \mathrm{sec}$ is
$\mathrm{v}_{\mathrm{av}}=\frac{\Delta \mathrm{x}}{\Delta \mathrm{t}}=\frac{\sqrt{(40)^{2}+(-10)^{2}}}{4}$
$\mathrm{v}_{\mathrm{av}}=\frac{5}{2} \sqrt{17} \mathrm{~m} / \mathrm{s}$
At $t=2 \sec \quad u=10 \times 2=20 \mathrm{~m} / \mathrm{s}$
After $\quad t=2 \mathrm{sec}$
$v=u+a t$
$0=20-10 t$
$\mathrm{t}=2 \mathrm{sec}$.

Hence, at $t=4 \mathrm{sec}$. the particle is at its farthest
distance from the $y$-axis.
The particle is at farthest distance from $y$-axis at $t$ $\geq 4$. Hence the available correct choice is $t=4$.

## DPP NO. - 20

1. If speed of a particle changes, the velocity of the particle definitely changes and hence the acceleration of the particle is nonzero.
Velocity of a particle change without change in speed.
When speed of a particle varies, its velocity cannot be constant.
2. $V_{w}=1 \hat{i}+1 \hat{j}$


$V=$ at
$\mathrm{V}=(0.2) 10$
$=2 \mathrm{~m} / \mathrm{sec}$.
$V_{\text {boat }}=2 \hat{i}+2 \hat{j}$
$V_{\text {wboat }}=V_{w}-V_{\text {boat }}$
$V_{\text {wboat }}=(1 \hat{i}+1 \hat{j})-(2 \hat{i}+2 \hat{j})=-1 \hat{i}-1 \hat{j}$
So, the flag will flutter towards south-west.
3. The retardation is given by

$$
\frac{d v}{d t}=-a v^{2}
$$

integrating between proper limits
$\Rightarrow-\int_{u}^{v} \frac{d v}{v^{2}}=\int_{0}^{t} a d t \quad$ or $\quad \frac{1}{v}=a t+\frac{1}{u}$
$\mathbf{t}=\sqrt{\frac{2 \ell}{\mathrm{a}}}$ required time after which ball hit the corner.
$\Rightarrow \frac{\mathrm{dt}}{\mathrm{dx}}=\mathrm{at}+\frac{1}{\mathrm{u}} \Rightarrow \mathrm{dx}=\frac{\mathrm{udt}}{1+\mathrm{aut}}$
integrating between proper limits
$\Rightarrow \quad \int_{0}^{\mathrm{s}} \mathrm{dx}=\int_{0}^{\mathrm{t}} \frac{\mathrm{udt}}{1+\mathrm{aut}} \Rightarrow \mathrm{S}=\frac{1}{\mathrm{a}} \ln (1+\mathrm{aut})$
4. $V=a+b x$
( $V$ increases as $x$ increases)
$\frac{\mathrm{dV}}{\mathrm{dt}}=\mathrm{b} \frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{bV}$
hence acceleration increases as V increases with x .
6. $\vec{v}=-\hat{i}+\hat{j}+2 \hat{k}$
$\vec{a}=3 \hat{i}-\hat{j}+\hat{k}$
$\vec{a} \cdot \vec{v}=-3-1+2<0$
hence $\theta>90^{\circ}$ between $\vec{a}$ and $\vec{v}$
so speed is decreasing
$\vec{a} \cdot \vec{v}=-3-1+2<0$
7. Solving the problem in the frame of train. Taking origin as corner 'B'
Along $x$ axis $x$ -
$x=u \cos \theta t \ldots .(1)$
Along y axis $y$ -
$y=u_{y} t+\frac{1}{2} a_{y} t^{2}$

$0=u \sin \theta t-\frac{1}{2} a t^{2}$
As the ball is thrown towards ' D '
$\tan \theta=\frac{\ell}{\mathrm{x}}$
From equation (1), (2) \& (3) we get
8. At position A balloon drops first particle So, $u_{A}=0, a_{A}=-g, t=3.5 \mathrm{sec}$.
$S_{A}=\left(\frac{1}{2} g t^{2}\right)$
Balloon is going upward from $A$ to $B$ in 2 sec.so distance travelled by balloon in 2 second.
$\left(S_{B}=\frac{1}{2} a_{B} t^{2}\right)$
$a_{B}=0.4 \mathrm{~m} / \mathrm{s}^{2} \quad, \quad \mathrm{t}=2 \mathrm{sec}$.
$S_{1}=B C=(S B+S A)$
Distance travell by second stone which is droped from balloon at $B$
$\mathrm{u}_{2}=\mathrm{u}_{\mathrm{B}}=\mathrm{a}_{\mathrm{B}} \mathrm{t}=0.4 \times 2=0.8 \mathrm{~m} / \mathrm{s}$
$\mathrm{t}=1.5 \mathrm{sec}$.

$$
\begin{equation*}
\left(S_{2}=u_{2} t-\frac{1}{2} g t^{2}\right) \tag{iv}
\end{equation*}
$$



Distance between two stone
$\Delta \mathrm{S}=\mathrm{S}_{1}-\mathrm{S}_{2}$.

## DPP NO. - 21

1. 


$Q$ measures acceleration of $P$ to be zero.
$\therefore Q$ measures velocity of $P$, i.e. $\vec{v}_{P Q}$, to be constant. Hence $Q$ observes $P$ to move along straight line.
$\therefore$ For P and Q to collide Q should observe P to move along line PQ .
Hence PQ should not rotate.
2. Let initial and final speeds of stone be $u$ and $v$.
$\therefore \quad \mathrm{v}^{2}=\mathrm{u}^{2}-2 \mathrm{gh}$
and $v \cos 30^{\circ}=u \cos 60^{\circ}$
solving 1 and 2 we get $u=\sqrt{3 g h}$
3. Flag will flutter in the direction of wind and opposite to the direction of velocity of man
i.e. in the direction of $\mathrm{V}_{\mathrm{wm}}$

4. (i)


$$
a=0
$$


$N=F$.
(ii)

$a=\frac{2 F}{4 m}=\frac{F}{2 m}$

$F-N=m a$
$N=F-m\left(\frac{F}{2 m}\right)=\frac{F}{2}$.
(iii)

$a=\frac{3 F}{4 m}$

$\mathrm{F}-\mathrm{N}=\mathrm{ma}$
$\mathrm{N}=\mathrm{F}-\mathrm{ma}$
$N=F-m\left(\frac{3 F}{4 m}\right)$
$N=\frac{F}{4}$.
(iv)

$a=\frac{3 F}{4 m}$

$2 F-N=m a \quad N=2 F-m\left(\frac{3 F}{4 m}\right)$
$N=\frac{5 F}{4}$.
(v)

$a=\frac{3 F}{3 m}=\frac{F}{m}$

$N+F=m a \quad N+F=m\left(\frac{F}{m}\right)$
$N=0$.
5. F.B.D. of block
$N^{2}=F^{2}+(m g)^{2}$

$N=10 \sqrt{2} N$
6. $\mathrm{AB}=2 \mathrm{R} \cos \theta$
acceleration along $A B$
$a=g \cos \theta$
$u=0$ from $A$ to $B$
$S=u t+\frac{1}{2} a t^{2}$

$2 R \cos \theta=0+\frac{1}{2}(g \cos \theta) t^{2}$
$t=2 \sqrt{\frac{R}{g}}$
7. Unit vector in direction of $(1,0,0)$ to $(4,4,12)$ is

$$
\frac{(4-1) \hat{i}+(4-0) \hat{j}+(12-0) \hat{k}}{13}
$$

Hence position of particle at $t=2 \mathrm{sec}$ is :
$\vec{r}_{f}=\vec{r}_{i}+\vec{v} \times 2=31 \hat{i}+40 \hat{j}+120 \hat{k}$
8. $\quad \mathrm{a}=\frac{\mathrm{F}}{\mathrm{m}} \quad \mathrm{V}^{2}=\mathrm{u}^{2}+2 \mathrm{as} \quad(\mathrm{u}=0)$

$$
V \propto \sqrt{2\left(\frac{F}{m}\right) S} \quad V \propto \frac{1}{\sqrt{m}}
$$

## DPP NO. - 22

1. From geometry :
$\cos \theta=\frac{3}{5}$
$\sin \theta=\frac{4}{5}$


As sphere is at equilibrium,
$\mathrm{T} \sin \theta=\mathrm{w}$
$T\left(\frac{4}{5}\right)=w$
$T=\frac{5 w}{4}$.
2. Resolving forces at point $A$ along string $A B$

3. $v=0 \Rightarrow x^{2}-5 x+4=0$
$x=1 m \& 4 m$
$\frac{d v}{d t}=(2 x-5) v=(2 x-5)\left(x^{2}-5 x+4\right)$
at $\mathrm{x}=1 \mathrm{~m}$ and $4 \mathrm{~m} ; \frac{\mathrm{dv}}{\mathrm{dt}}=0$
4. $a=\left(\frac{5-4}{5+4}\right) g=\frac{g}{9}$
$\mathrm{T}-\mathrm{mg}=\mathrm{ma}$

$T=m(g+a)$
$=1\left(g+\frac{g}{9}\right)=\frac{10 g}{9}$.
5. Time taken by ball from $O$ to $A$ is same as that from $A$ to $B$.

$10=15 t-\frac{1}{2}(10) t^{2}$
$5 t^{2}-15 t-10=0$
$\mathrm{t}^{2}-3 \mathrm{t}-2=0$
$\mathrm{t}=1,2$
$t=2$ is invalid as it is the time taken by the ball to come at A' if there was no roof.
$\therefore \mathrm{t}=1$ seconds.

During this the ball will travel $\mathrm{V} \times \mathrm{t}=20 \times 2$ $=40 \mathrm{~m}$ on the floor.
6.

$r=5 \mathrm{~cm} ; \mathrm{R}=8 \mathrm{~cm}$
FBD of sphere 1

$\mathrm{N}_{1}=\mathrm{W}+\mathrm{N}_{3} \sin \theta$
$N_{2}=N_{3} \cos \theta$
FBD of sphere 2

$A C=2 R-2 r$
$A B=2 r$
$\cos \theta=\frac{A C}{A B}=\frac{R-r}{r}$
$\mathrm{N}_{4}=\mathrm{N}_{3} \cos \theta$
$\mathrm{W}=\mathrm{N}_{3} \sin \theta$

Ans. $\mathrm{N}_{4}=\mathrm{W} \cot \theta$
$\mathrm{N}_{3}=\mathrm{W} \operatorname{cosec} \theta$
$\mathrm{N}_{2}=\mathrm{W} \cot \theta$
$\mathrm{N}_{1}=2 \mathrm{~W}$.
7. $\Rightarrow 0.2 \mathrm{~g}=0.7 \mathrm{a}$
$\Rightarrow \mathrm{a}=\frac{2 \mathrm{~g}}{7} \mathrm{~m} / \mathrm{s}^{2}$
For the case, it comes to rest when $\mathrm{V}=0$
$0=7+\left(-\frac{2 g}{7}\right) \mathrm{t} \Rightarrow \mathrm{t}=\frac{49}{2 \mathrm{~g}}=2.5 \mathrm{~s}$


Distance travelled till it comes to rest
$0=7^{2}+2\left(-\frac{2 g}{7}\right) s$
$\mathrm{S}=8.75 \mathrm{~m}$
So in next 2.5 s , it covers 8.75 m towards right.
Total distance $=2 \times 8.75=17.5 \mathrm{~m}$
After 5 s , it speed will be same as that of initial (7 $\mathrm{m} / \mathrm{s}$ ) but direction will be reversed.
8. Acceleration of system $a=\frac{F}{m_{A}+m_{B}+m_{C}}$
$a=\frac{60}{10+20+30}=1 \mathrm{~m} / \mathrm{s}^{2}$
FBD of $A$ :

$T_{1}=m_{A} \cdot a$
$\mathrm{T}_{1}=10(1)=10 \mathrm{~N}$
FBD of $B$ :

$\mathrm{T}_{2}-\mathrm{T}_{1}=\mathrm{m}_{\mathrm{B}} \mathrm{a}$
$\mathrm{T}_{2}-10=20(1)$
$\mathrm{T}_{2}=30 \mathrm{~N}$.

## DPP NO. - 23

1. for (man + platform) system :
$2 \mathrm{mg}-4 \mathrm{~T}=2 \mathrm{~m}(\mathrm{a})$

$\Rightarrow 2 m g-4\left(\frac{\mathrm{mg}}{2}\right)=2 \mathrm{~m}(\mathrm{a})\left[\because \mathrm{T}=\frac{\mathrm{mg}}{2}\right]$
$\Rightarrow \mathrm{a}=0$
2. Let $a=$ acceleration of $m_{1}$
then acceleration of pulley $=\frac{a+0}{2}=\frac{a}{2}$
If acceleration of $m_{2}=b$
Then $\quad 0+\frac{b}{2}=\frac{a}{2}$
Hence $a=b$
$\mathrm{T}=\mathrm{m}_{1} \mathrm{a}, \mathrm{m}_{2} \mathrm{~g}-\mathrm{T}=\mathrm{m}_{2} \mathrm{a}$
$\therefore \quad a=\frac{m_{2} g}{m_{1}+m_{2}}$
3. Method - I

As cylinder will remains in contact with wedge A
$\mathrm{V}_{\mathrm{x}}=2 \mathrm{u}$


As it also remain in contact with wedge $B$ $\mathrm{u} \sin 30^{\circ}=\mathrm{V}_{\mathrm{y}} \cos 30^{\circ}-\mathrm{V}_{\mathrm{x}} \sin 30^{\circ}$
$\mathrm{V}_{\mathrm{y}}=\mathrm{V}_{\mathrm{x}} \frac{\sin 30^{\circ}}{\cos 30^{\circ}}+\frac{\mathrm{U} \sin 30^{\circ}}{\cos 30^{\circ}}$
$\mathrm{V}_{\mathrm{y}}=\mathrm{V}_{\mathrm{x}} \tan 30^{\circ}+\mathrm{u} \tan 30^{\circ}$
$V_{y}=3 u \tan 30^{\circ}=\sqrt{3} u$
$V=\sqrt{V_{x}^{2}+V_{y}^{2}}=\sqrt{7} \mathbf{u}$ Ans.

## Method - II

In the frame of $A$

$3 u \sin 30^{\circ}=V_{y} \cos 30^{\circ}$
$\Rightarrow \mathrm{V}_{\mathrm{y}}=3 \mathrm{u} \tan 30^{\circ}=\sqrt{3} \mathrm{u}$
and $O V_{x}=2 u$
$\Rightarrow \mathrm{V}=\sqrt{\mathrm{V}_{\mathrm{x}}^{2}+\mathrm{V}_{\mathrm{y}}^{2}}=\sqrt{7} \mathrm{u}$ Ans.
4. $\ell_{1}+2 \ell_{2}=$ constant
$\therefore \quad \frac{\mathrm{d} \ell_{1}}{\mathrm{dt}}+\frac{2 \mathrm{~d} \ell_{2}}{\mathrm{dt}}=0$

$(5+5)+2\left(5+v_{B}\right)=0$ or $v_{B}=10 \mathrm{~m} / \mathrm{s}$
5. Assume that acceleration of particle is $a_{p}$ and acceleration of wedge is $a_{w}$
Then, $\quad a_{w}=g \sin \theta$
From wedge constant
$a_{p}=a_{w} \sin \theta=g \sin ^{2} \theta$
$h=\frac{1}{2} g \sin ^{2} \theta t^{2}$
$t=\sqrt{\frac{2 h}{g \sin ^{2} \theta}}$.
6. From Newtons third law, the force exerted by table on block is equal to that exerted by block on the
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table. Therefore block exerts a 10 N force on table. Since the upward force on the block is larger than downward force, it moves upwards.
7.


FBD of block $M_{2}=2 k g$

$20-T=2 a$

FBD of block $M_{1}=3 k g$

$=3 \times \frac{10 \sqrt{3}}{2}$
$=15 \sqrt{3} \mathrm{~N}$.
$=15 \mathrm{~N}$
$\mathrm{T}-15=3 \mathrm{a}$
(i) + (ii)
$5=5 a$
$\Rightarrow \quad a=1 \mathrm{~m} / \mathrm{s}^{2} \quad ; \quad T=18 \mathrm{~N}$.
8. (i) $a=\frac{2 m g-m g}{3 m}=\frac{g}{3}$
(ii) $a=\frac{2 m g-m g}{m}=g$
(iii) $a=\frac{2 m g}{m}=2 g$
(iv) $a=\frac{2 g}{3}$

## DPP NO. - 24

$\frac{d x}{d t}=8 y \frac{d y}{d t}$
$\mathrm{V}_{\mathrm{x}}=8 \mathrm{y} \mathrm{V}_{\mathrm{y}}$
$\mathrm{V}_{\mathrm{x}}=4$
$a_{x}=0$
$0=a_{x}=8\left[y \cdot a_{y}+V^{2}{ }_{y}\right]$

$-y a_{y}=V_{y}^{2}$
$\left|a_{y}\right|=\frac{v_{y}^{2}}{y}$
$\left|a_{y}\right|=\frac{v_{x}^{2}}{64 y^{3}}=\frac{16}{64 \times y^{3}}$
at $y=1 \Rightarrow \quad\left|a_{y}\right|=\frac{1}{4}$
$y=1 \quad \Rightarrow \quad\left|a_{y}\right|=\frac{1}{4}$
5.


Relative between $\mathrm{a}_{1}$ \& $\mathrm{a}_{2}$
$\mathrm{a}_{1}=2 \mathrm{a}_{2}=2 \mathrm{a}$
Relative between $\mathrm{T}_{1} \& \mathrm{~T}_{2}$
$\mathrm{T}_{2}=2 \mathrm{~T}_{1}=2 \mathrm{~T}$
$\mathrm{T}_{1}=\mathrm{M}_{1} \mathrm{a}_{1}$
$M_{2} g-T_{2}=M_{2} a_{2}$
$2 \mathrm{~T}=4 \mathrm{M}_{1} \mathrm{a}$
$M_{2} g-2 T=M_{2} a$
$M_{2} g=a\left(4 M_{1}+M_{2}\right) \Rightarrow a=\frac{M_{2} g}{4 M_{1}+M_{2}}$.
6.


18 kg
$3 F=180$
$F=60 \mathrm{~N}$
$\mathrm{T}=4 \mathrm{~F} \quad=210 \mathrm{~N}$
Force balance on system

$$
\begin{aligned}
& T=F+180 \\
& T=60+180=240 N .
\end{aligned}
$$

7. False There acceleration may be different.

True $\mathrm{T}=\frac{\mathrm{W}}{\mathrm{V}}$ to minimize $\mathrm{T}, \mathrm{V}$ will be maximum.
i.e whole effort of swimmer must towards opposite bank.
8. (i) FBD of 2 kg

$\mathrm{N}_{23}-20=2(2)$
$\mathrm{N}_{23}=24 \mathrm{~N}$
$\mathrm{N}_{23}=24 \mathrm{~N}$
(ii) FBD of 3 kg
$\mathrm{N}_{34}-\mathrm{N}_{23}-30=3(2)$

$N_{34}=N_{23}+30+6$
$\mathrm{N}_{34}=24+30+6=60 \mathrm{~N}$
FBD of 4 kg
$N_{G}-N_{34}-40=4(2)$


$$
\begin{aligned}
& \mathrm{N}_{\mathrm{G}}=\mathrm{N}_{34}+40+8 \\
& \mathrm{~N}_{\mathrm{G}}=60+40+8=108 \mathrm{~N}
\end{aligned}
$$

